Improved Cryptanalysis of Py

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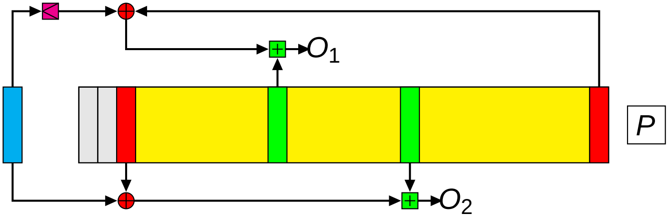
State of the Art in Stream Ciphers 2006

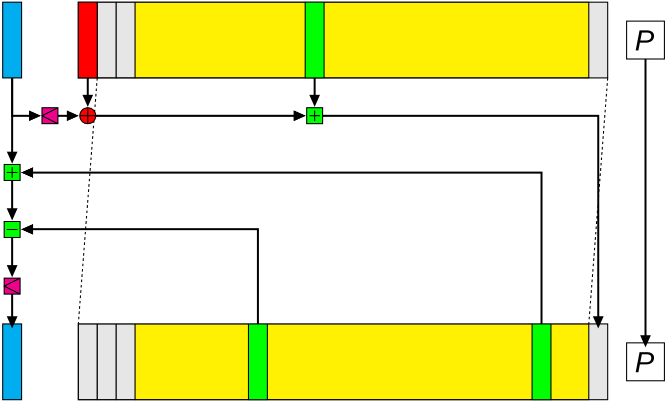
Py

- eSTREAM entrant by Eli Biham and Jennifer Seberry
- Fast in software (2.6 cycles/byte on some platforms)
- SPP attack: 2^{88} bytes of output
- Our attack: 2^{72} bytes

Output



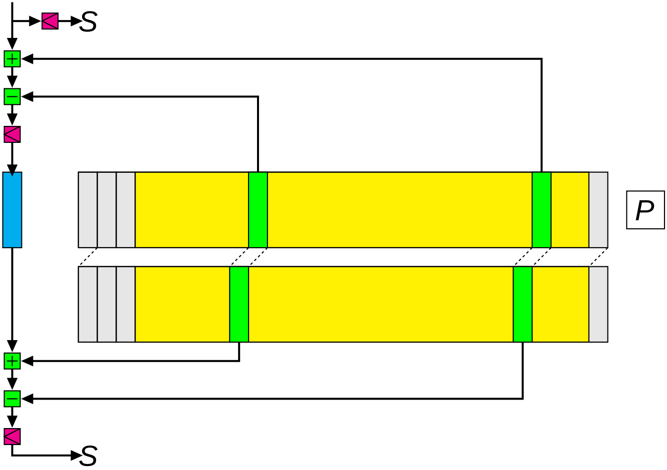
Update



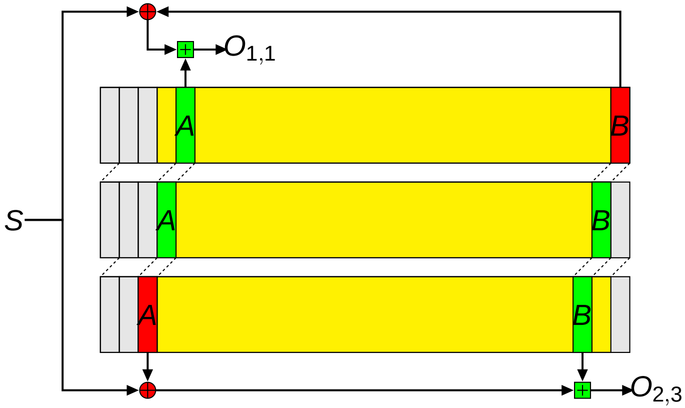
SPP attack

- Gautham Sekar, Souradyuti Paul, Bart Preneel
- Defines event L with $\Pr[L] \approx 2^{-41.91}$
- When L occurs, two output bits are the same

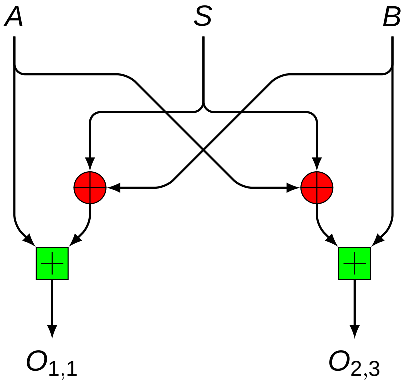
Event L (1)



Event L (2)



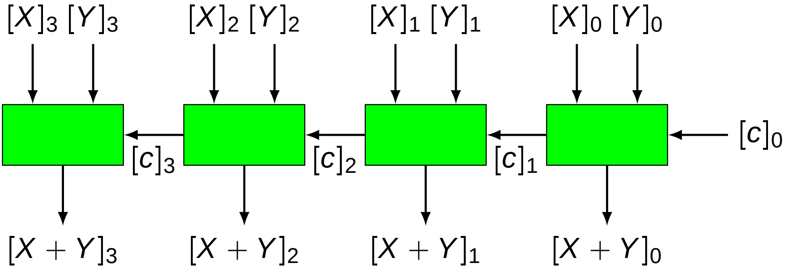
Result of event L



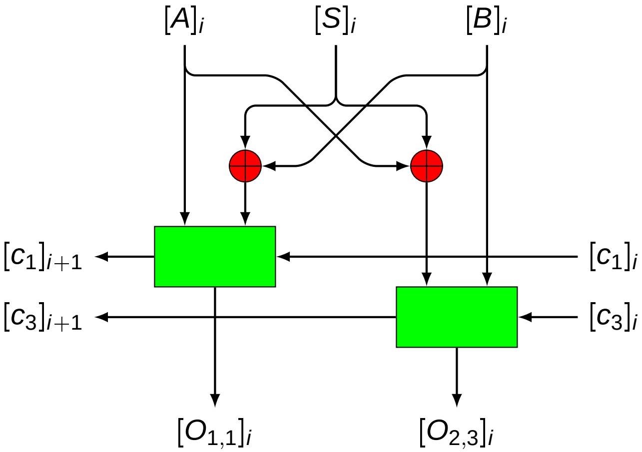
Improving on the attack

- Use all bits of $O_{1,1}, O_{2,3}$
- Group output by column bitwise
- Find exact probability $\Pr[O_{1,1}, O_{2,3} = o_{1,1}, o_{2,3} | L]$
- Apply optimal distinguisher

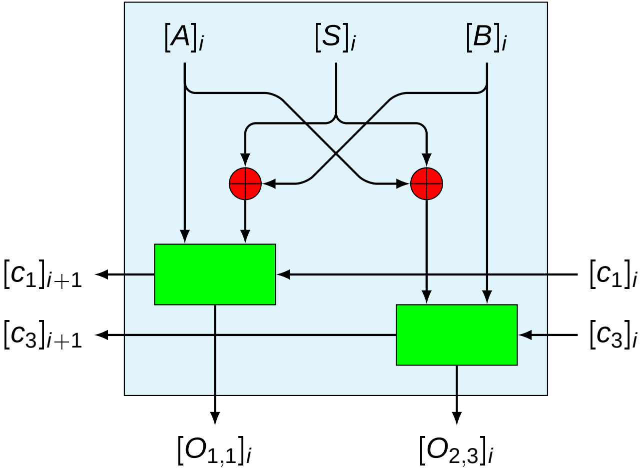
Addition



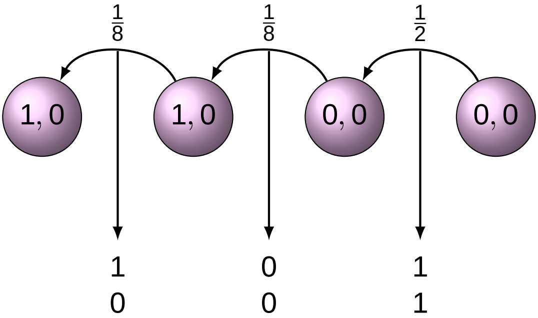
Carry propagation



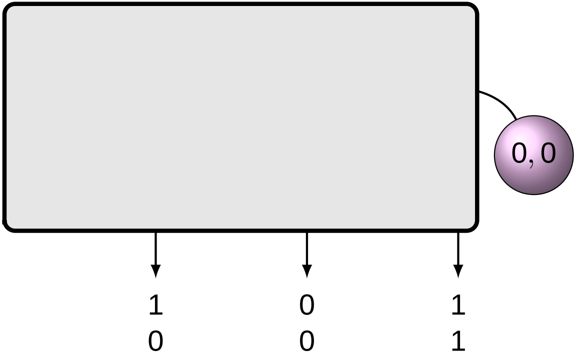
Carry propagation



Hidden Markov model



Hidden Markov model



The forward algorithm

$$\Pr\left[\begin{array}{ccc} 1 & 0 & 1 \\ 0 & 0 & 1 \end{array}\right] = \mathbf{1}_{1\times 4} M_{1,0} M_{0,0} M_{1,1} \pi_0$$

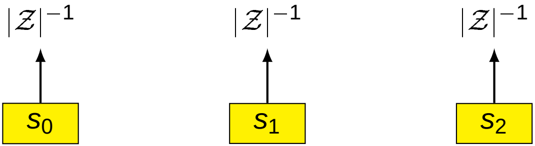
where $\mathbf{1}_{1\times 4} = \left(\begin{array}{cccc} 1 & 1 & 1 & 1 \end{array} \right)$ and $\pi_0 = \left(\begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \end{array} \right)$

Optimal distinguisher

- Thomas Baignères, Pascal Junod, Serge Vaudenay
- Optimal distinguisher chooses the distribution which has the highest probability of producing the observed output

Optimal distinguisher

Optimal distinguisher



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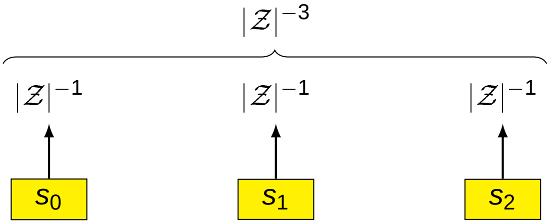

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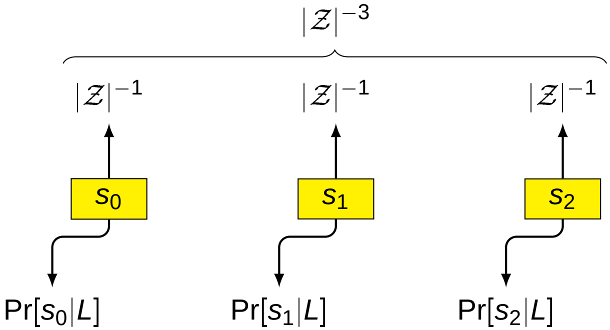

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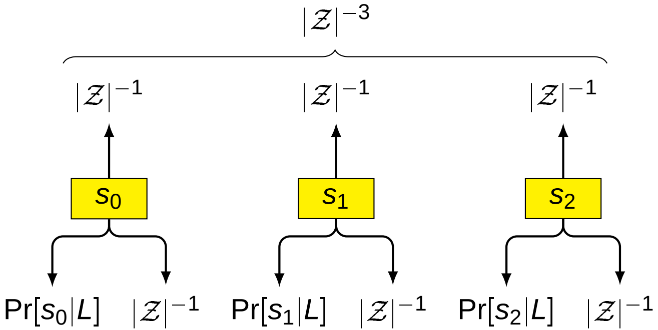

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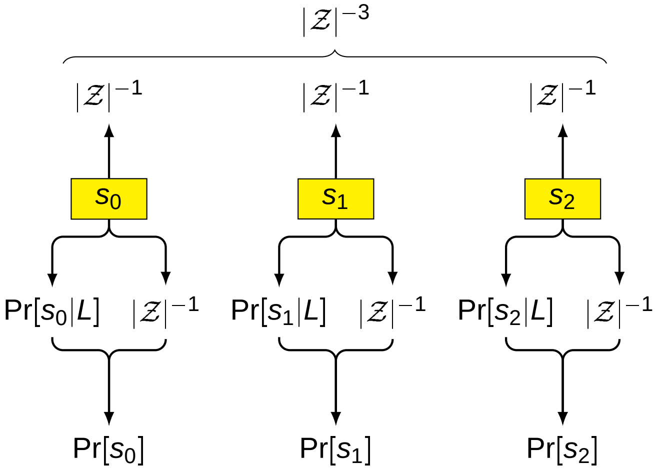

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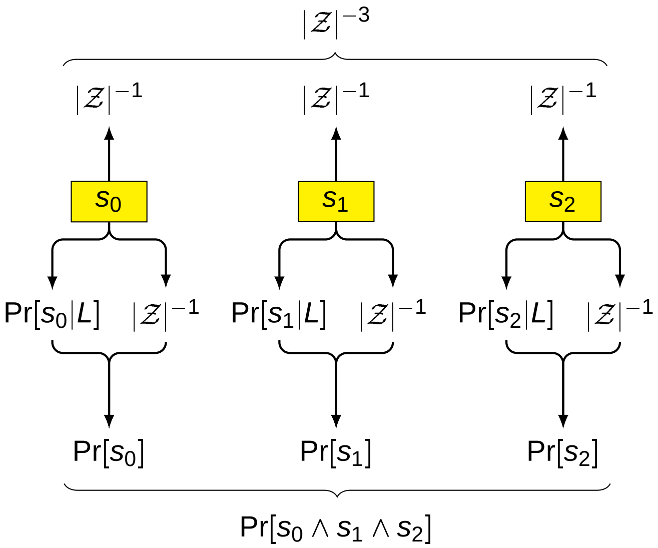

Efficacy of optimal distinguisher

- Where distribution is “close” to uniform random, efficacy $\beta = |\mathcal{Z}| \sum_{z \in \mathcal{Z}} \left(\Pr[z] - \frac{1}{|\mathcal{Z}|} \right)^2$

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- SPP attack: $\beta = \Pr[L]^2$ so around 2^{85} samples

Efficacy of our distinguisher

$$\sum_{z \in \mathcal{Z}} \Pr[z \mid L]^2$$

Efficacy of our distinguisher

$$\sum_{z \in \mathcal{Z}} \Pr[z|L]^2$$
$$= \sum (\mathbf{1}_{1\times 4} M_{31} M_{30} \dots M_0 \pi_0)^2$$

$$M_i \in \{M_{0,0}, M_{0,1}, M_{1,0}, M_{1,1}\}$$

Efficacy of our distinguisher

$$
\begin{aligned}
& \sum_{z \in \mathcal{Z}} \Pr[z|L]^2 \\
= & \sum \left(\mathbf{1}_{1\times 4} M_{31} M_{30} \ldots M_0 \pi_0\right)^2 \\
= & \sum \left(\mathbf{1}_{1\times 4} M_{31} M_{30} \ldots M_0 \pi_0\right)\left(\mathbf{1}_{1\times 4} M_{31} M_{30} \ldots M_0 \pi_0\right)^T
\end{aligned}
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= & \sum \left(\mathbf{1}_{1\times 4} M_{31} M_{30} \ldots M_0 \pi_0 \pi_0^T M_0^T \ldots M_{30}^T M_{31}^T \mathbf{1}_{1\times 4}^T\right)
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= & \sum \left(\mathbf{1}_{1\times 4} M_{31} M_{30} \ldots M_0 \pi_0 \pi_0^T M_0^T \ldots M_{30}^T M_{31}^T \mathbf{1}_{1\times 4}^T\right) \\
= & \mathbf{1}_{1\times 4} \sum \left(M_{31} M_{30} \ldots M_0 \pi_0 \pi_0^T M_0^T \ldots M_{30}^T M_{31}^T\right) \mathbf{1}_{1\times 4}^T
\end{aligned}
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$$M_i \in \{M_{0,0}, M_{0,1}, M_{1,0}, M_{1,1}\}$$

Efficacy of our distinguisher

$$H_i = \sum M_{i-1} M_{i-2} \ldots M_1 M_0 \pi_0 \pi_0^T M_0^T M_1^T \ldots M_{i-2}^T M_{i-1}^T$$

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H_0 &= \pi_0 \pi_0^T
\end{aligned}
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H_0 &= \pi_0 \pi_0^T \\
H_{i+1} &= \sum_{M \in \{M_{0,0}, M_{0,1}, M_{1,0}, M_{1,1}\}} M H_i M^T
\end{aligned}
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H_{i+1} &= \sum_{M \in \{M_{0,0}, M_{0,1}, M_{1,0}, M_{1,1}\}} M H_i M^T \\
\beta &= \Pr[L]^2 \left(2^{64} \left(\mathbf{1}_{1\times 4} H_{32} \mathbf{1}_{1\times 4}^T \right) - 1 \right)
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$$

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\begin{aligned}
H_i &= \sum M_{i-1} M_{i-2} \dots M_1 M_0 \pi_0 \pi_0^T M_0^T M_1^T \dots M_{i-2}^T M_{i-1}^T \\
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\beta &= \Pr[L]^2 \left(2^{64} \left(\mathbf{1}_{1\times 4} H_{32} \mathbf{1}_{1\times 4}^T \right) - 1 \right) \\
&\approx 60552 \Pr[L]^2
\end{aligned}
$$

Conclusions

- We can efficiently calculate the efficacy of HMM-based distinguishers
- Distinguisher advantage is 0.53 given 2^{64} bytes from 2^8 key/IV pairs
- Advantage is 0.03 given a single 2^{64}-byte stream
- Can this be improved still further?

http://www.ciphergoth.org/crypto/py